

$$f(x) = \frac{(x+1)^2}{x^2+1}$$

a) Find domain, extrema, c.p. & infl. pts

Domain is all real numbers since $x^2+1 \neq 0$

$$f'(x) = \frac{-2x^2+2}{(x^2+1)^2} = \frac{-2(x+1)(x-1)}{(x^2+1)^2}$$

$f'(x)$ exists everywhere

$$f'(x) = 0 \text{ at } x=1, -1$$

$$f(1) = \frac{4}{2} = 2, \quad f(-1) = 0$$

c.p. $(1, 2)$ & $(-1, 0)$

$$f' \quad \begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \\ | \quad \quad | \\ x \quad -1 \quad \quad 1 \end{array}$$

$$\lim_{x \rightarrow \infty} f(x) = 1, \quad \lim_{x \rightarrow -\infty} f(x) = 1$$

Abs. min. at $(-1, 0)$
Abs. max at $(1, 2)$

$$f''(x) = \frac{4x(x^2-3)}{(x^2+1)^3}$$

$f''(x)$ exists everywhere

$$f''(x) = 0 \text{ at } x=0, \sqrt{3}, -\sqrt{3}$$

$$f'' \quad \begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \\ | \quad \quad | \quad \quad | \\ x \quad -\sqrt{3} \quad \quad 0 \quad \quad \sqrt{3} \end{array}$$

$$f(-\sqrt{3}) = 1 - \frac{\sqrt{3}}{2}$$

$$f(0) = 1$$

$$f(\sqrt{3}) = 1 + \frac{\sqrt{3}}{2}$$

Infl. pts. at $(-\sqrt{3}, 1 - \frac{\sqrt{3}}{2})$
 $(0, 1)$
 $(\sqrt{3}, 1 + \frac{\sqrt{3}}{2})$

b) Find VA, HA & SA

No VA or SA

$$\lim_{x \rightarrow \infty} f(x) = 1$$

$$\lim_{x \rightarrow -\infty} f(x) = 1$$

so HA at $y=1$

c) Find incr., decr., concave \uparrow & concave \downarrow intervals

$$f' \begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \\ | \quad | \quad | \quad | \\ x \quad -1 \quad \quad \quad 1 \end{array}$$

Incr. on $(-1, 1)$

Decr. on $(-\infty, -1) \cup (1, \infty)$

$$f'' \begin{array}{c} - \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \\ | \quad | \quad | \quad | \quad | \\ x \quad -\sqrt{3} \quad 0 \quad \sqrt{3} \end{array}$$

C. \uparrow on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

C. \downarrow on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

d) Graph

