Math 217 - Interpreting the Standard Deviation (Supplement to Module 2.4.1)

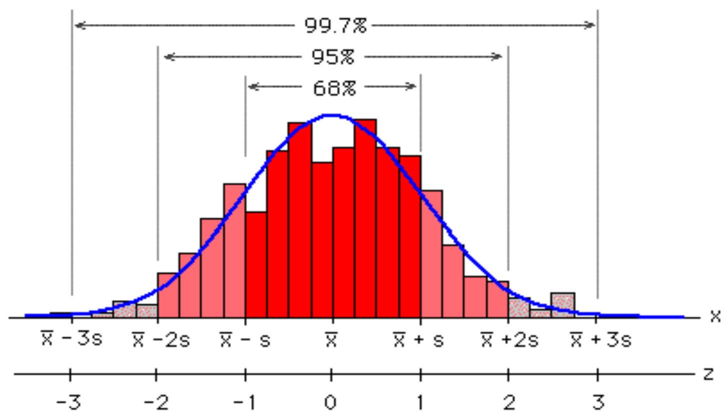
Empirical Rule

A student asked an instructor about the distribution of exam scores after she saw her score of 87 out of 100. The instructor told her the distribution of test scores was approximately a bell-shaped with a mean score of 75 and a standard deviation of 10.

Most people have an intuitive grasp of the mean score as being the "average student's score" and would say this student did better than average. However, having an intuitive grasp of standard deviation is more challenging. The Empirical Rule is a helpful tool in explaining standard deviation.

The standard deviation is a measure of variability or spread from the center of the data as defined by the mean. The empirical rules states that for bell-shaped data:

> 68% of the data is within 1 standard deviation of the mean.
>
> 95% of the data is within 2 standard deviations of the mean.
>
> 99.7% of the data is within 3 standard deviations of the mean.



In the exam score example where the sample mean $\bar{x} = 75$ points and the sample standard deviation $s = 10$, the interpretation would be:

68% of students scored between 65 and 85 (75 plus or minus 10).
95% of students scored between 55 and 95 (75 plus or minus 20).
99.7% of students scored between 45 and 105 (75 plus or minus 30).

The student who scored an 87 would be in the upper 16% of the class, more than one standard deviation above the mean score.

The z-score

Related to the Empirical Rule is the z-score which measures how many standard deviations a particular data point is above or below the mean. Unusual observations would have a z-score over 2 or under -2. Extreme observations would have z-scores over 3 or under -3 and should be investigated as potential outliers.

Formula for z-score: $$z - score = \frac{x - \bar{x}}{s}$$

For the student who received an 87 on the exam we can calculate the Z-score: $z - score = \frac{87-75}{10} = 1.2$

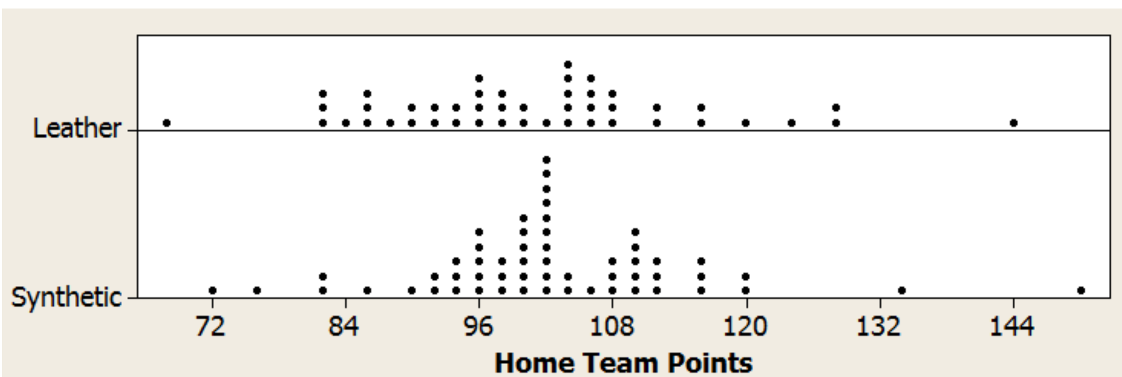
The z-score of 1.2 tells us the student's score was well above average, but not highly unusual.

Interpreting z-score for Several Students

Test Score	z-score	Interpretation
87	+1.2	well above average
71	-0.4	slightly below average
99	+2.4	unusually above average
39	-3.6	extremely below average

Try These:

The dot plot shows home team scores from a sample of games from the 2006-2007 NBA season.



1. For the games when the new synthetic ball was used, the sample mean was 102.1 points and the standard deviation was 12.4 points.

    a. Use the Empirical Rule to determine the ranges of score that occur 68% of the time.

    b. Use the Empirical Rule to determine the ranges of score that occur 95% of the time.

    c. Use the Empirical Rule to determine the ranges of score that occur 99.7% of the time.

2. For the games when the original leather ball was used, the sample mean was 100.5 points and the standard deviation was 12.1 points.

   a. Use the Empirical Rule to determine the ranges of score that occur 68% of the time.

   b. Use the Empirical Rule to determine the ranges of score that occur 95% of the time.

   c. Use the Empirical Rule to determine the ranges of score that occur 99.7% of the time.

3. Determine the z-score for a home team that scores 92 points with the synthetic ball. Is this result unusual?

4. Determine the z-score for a home team that scores 135 points with the leather ball. Is this result unusual?