

1. According to a U.S. Census Bureau report, 21.6% of US residents speak a language other than English at home. (Source: <https://cis.org/Report/655-Million-US-Residents-Spoke-Foreign-Language-Home-2016> This is significantly lower for the US nationwide than the reported statistic for California, reflecting the greater diversity in California's population.)

Suppose that a sample of 20 US residents were randomly selected, independent of each other.

$X$  = the number of California residents in the sample who speak a language other than English at home

a. [2 points] Write the distribution for  $X$  using the appropriate notation:  $X \sim B(20, .216)$

b. [1 point] If many groups of 20 US residents were surveyed, on average, how many in each sample would be expected to speak a language other than English at home.  
State answer to 2 decimal places (hundredths).

$$\mu = np = 20(.216) = 4.32$$

**For parts c, d, e:**

Show your work by writing the calculator command and write your answer.

Round all probabilities to 3 decimal places, unless the probability is lower than .001.

If the probability is less than .001, then round the answer to 6 decimal places.

c. [1 point] Find the probability that at most 7 people in a group of 20 US residents speak a language other than English at home.

$$P(X \leq 7) = \text{binomcdf}(20, .216, 7) = .951$$

d. [2 points] Find the probability that at least 9 people in a group of 20 US residents speak a language other than English at home.

$$P(X \geq 9) = 1 - P(X \leq 8) = 1 - \text{binomcdf}(20, .216, 8) = .016527 \approx .017$$

e. [2 points] Find the probability that exactly 12 people in a group of 20 US residents speak a language other than English at home.

$$P(X = 12) = \text{binompdf}(20, .216, 12) = .000185$$

2. [3 points] A quiz contains 9 true-false questions. A student answers the questions by guessing randomly.

If a student answers less than 6 questions correctly, they will fail the quiz.

Find the probability that a student answers less than 6 questions correctly if randomly guessing.

Show your work by writing the calculator command and write your answer.

Round all probabilities to 3 decimal places.

$$X \sim B(9, .5)$$

$$P(X < 6) = P(X \leq 5) = \text{binomcdf}(9, .5, 5) = .746$$

(If randomly guessing the student has a .746 probability of failing the quiz)

3. [6 points] A venture capitalist has a choice of two start-up companies to invest in.
- Startup A has a 10% chance of returning a \$600,000 profit, a 50% chance of returning a \$75,000 profit, and a 40% chance of losing \$150,000.
  - Startup B has a 20% chance of returning a \$600,000 profit, a 30% chance of returning a \$200,000 profit, and a 50% chance of losing \$300,000.

For each investment, construct its PDF (probability distribution function) table. Then determine which investment (A or B) has the highest expected return, on average. You must show correct work for your calculations to justify your conclusion. No correct work = no credit, even if your conclusion is correct.

pdf table and expected return for investment A

X(\$)	P(X)
600000	.10
75000	.50
-150000	.40

$$\mu = 600000(.10) + 75000(.50) + (-150000)(.40)$$

$$\mu = \$37,500$$

pdf table and expected return for investment B

X(\$)	P(X)
600000	.20
200000	.30
-300000	.50

$$\mu = 600000(.2) + 200000(.3) + (-300,000)(.5)$$

$$\mu = \$30,000$$

For #3: Conclusion: Investment A has the highest expected return

4. [3 points] You play a game in which a standard 6 sided die is rolled once. If the number rolled is a 1, you win \$5. If the number rolled is even you win \$1. Otherwise you lose \$3. We are interested in the amount of money won or lost when playing this game.

If you played this game repeatedly, 150 times, what would be your expected total gain or loss?

X(\$)	P(X)
-3	2/6
1	3/6
5	1/6

$$\mu = \sum x P(x) = -3(2/6) + 1(3/6) + 5(1/6) = 1/3 = .3333... \approx .33$$

Answer is  $150\mu$ ; value depends on how you rounded the value for  $\mu$

$$150\left(\frac{1}{3}\right) = \$50 \quad 150(.333) = \$49.95 \quad 150(.33) = \$49.50$$