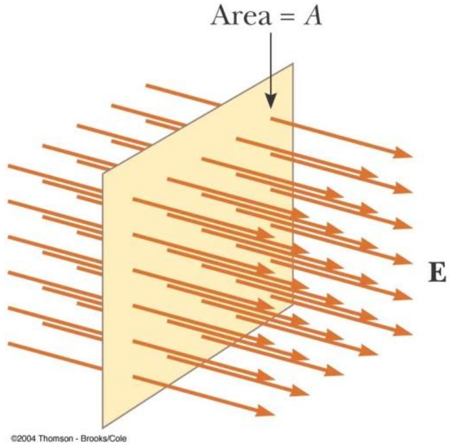
ELECTRIC FLUX

Consider a uniform **E**-field perpendicular to a surface area A.



Recall: Electric Field $E\ \alpha$ number of electric field lines per unit area.

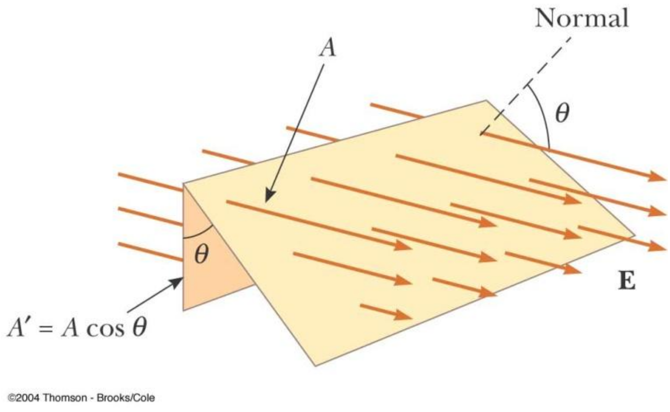
$\therefore\ EA\ \alpha$ number of E-field lines crossing the surface

$\boxed{\Phi = EA}$ Electric Flux

$[\Phi] = \frac{N}{C}m^2$

Electric Flux is proportional to the number of Electric Field Lines penetrating a surface.

If the E-field is not perpendicular to the surface area, then the flux will be less than EA because less electric field lines will penetrate A. Consider the wedge shape surface below. The electric field lines are perpendicular to the surface area A' but not to A.



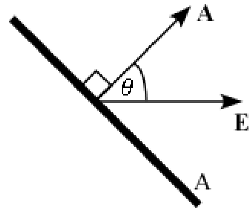
Since the same number of electric field lines cross both surfaces, the flux must be the same through both surfaces. The surface areas are related by:

$A' = A\cos\theta$

$\Phi = EA' = EA\cos\theta$

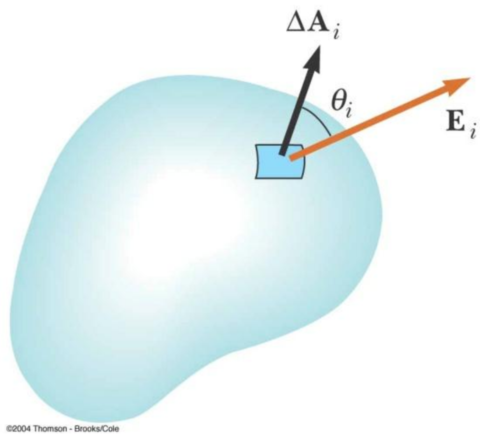
$\boxed{\Phi = EA\cos\theta = \vec{E}\bullet\vec{A}}$ Electric Flux

A = surface area vector $\perp$ to the surface area A



For a closed surface we define **A** to point in the outward direction. For an open surface you have two possible choices for the direction of **A** and thus you must specify its direction.

In general a surface may be curved and E will vary in magnitude and direction over the surface.



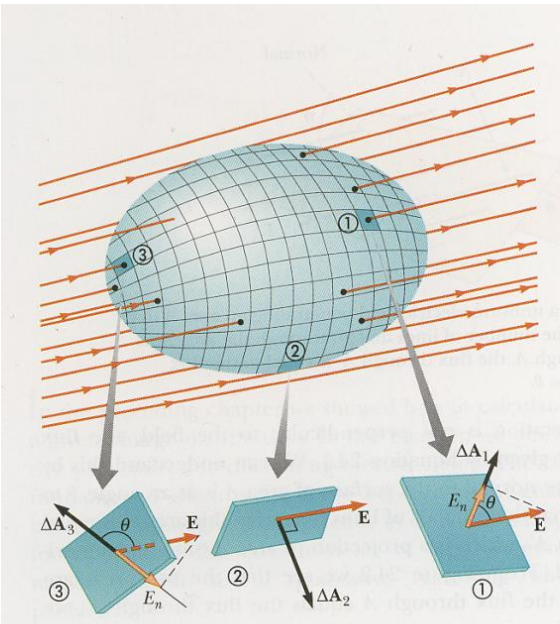
$$\Delta\Phi_i = \vec{E}_i \bullet \Delta\vec{A}_i$$

$$\Phi \approx \sum_i \Delta\Phi_i = \sum_i \vec{E}_i \bullet \Delta\vec{A}_i$$

$$\Phi = \lim_{\Delta\vec{A}_i \to 0} \sum_{i=1}^{\infty} \vec{E}_i \bullet \Delta\vec{A}_i$$

$$\boxed{\Phi = \int \vec{E} \bullet d\vec{A}}$$ Electric Flux

Recall that electric flux is proportional to the number of electric field lines crossing a surface. For a closed surface, electric field lines will be either entering or leaving the surface and thus the corresponding flux may be positive or negative.



1. The flux thru element A_1 is positive.
2. The flux thru element A_2 is zero.
3. The flux thru element A_3 is negative.

Thus, since the flux can be positive or negative for a closed surface, we talk about the net flux through a closed surface.

<u>For a closed surface, the net flux is proportional to the net number of electric file lines crossing the surface.</u>

<u>Φ = net number of electric field lines crossing a closed surface</u>

Where by net number we mean the number leaving the surface minus the number entering the surface. If more lines are leaving than entering the flux is positive and if more lines are entering than leaving the flux is negative.

For a closed surface the net electric flux is given by:

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$ Net Electric Flux Through a Closed Surface

Recall: The number of electric field lines entering or leaving a charge is directly proportional to the magnitude of the charge.

For the case in which we have a given charge distribution with a net charge bounded by closed surface we can extend this statement to be:

The net number of electric field lines through a closed surface surrounding a net charge is a qualitative measure of the net charge enclosed.

Q_{net} = net number of electric field lines crossing a closed surface

However, we also showed that:

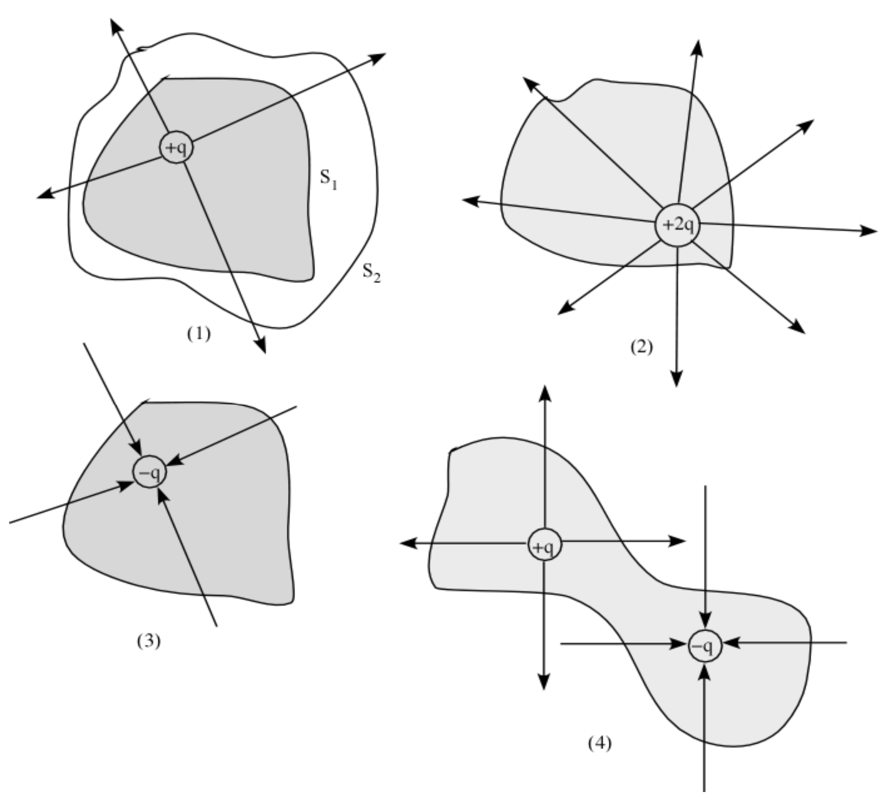
Φ = net number of electric field lines crossing a closed surface

Therfore,

$$\Phi \;\alpha\; Q_{net}$$

Thus the net flux through a closed surface surrounding a net charge is a qualitative measure of the net charge inside a closed surface.

This can be demonstrated by the following figures:



(1) The net flux across surface is +Φ because electric field lines are leaving surface. The net charge enclosed is +q.
(2) The net flux across surface is +2Φ because twice electric field lines are leaving surface. The net charge enclosed is +2q.
(3) The net flux across surface is -Φ because electric field lines are entering surface. The net charge enclosed is -q.
(4) The net flux across surface is Φ = 0 because the net number of electric field lines crossing the surface is zero. The net charge enclosed is 0.

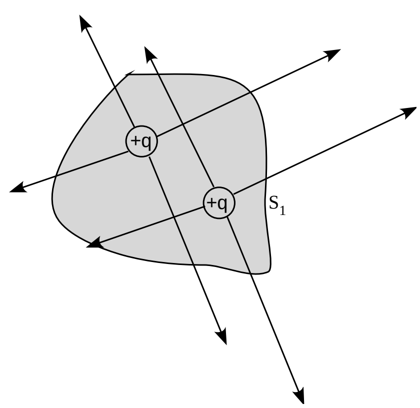
Three very important properties that these figures demonstrated are:

1. The net flux through a closed surface is a quantitative measure of the net charge inside a closed surface.

2. The net electric flux through any closed surface surrounding a net charge ‘q’ is independent of the shape of the surface.

3. The net electric flux is zero through any closed surface surrounding a zero net charge.

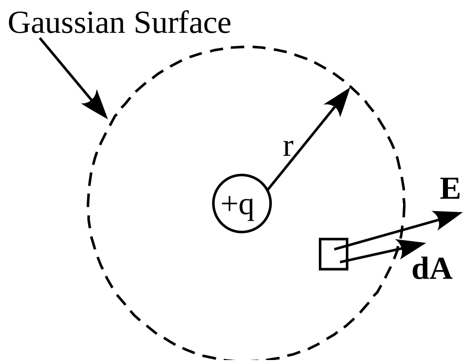
Therefore, if we know the net flux across a closed surface, then we know the net charge enclosed. Does this mean that we know the number of charges and their location?? NO!!!



Thus, we cannot determine the E-field at any point on the closed surface or any point in space. However, if the charge distribution and the closed surface are highly symmetric then one can find the E-field on a closed surface by using Gauss’s Law.

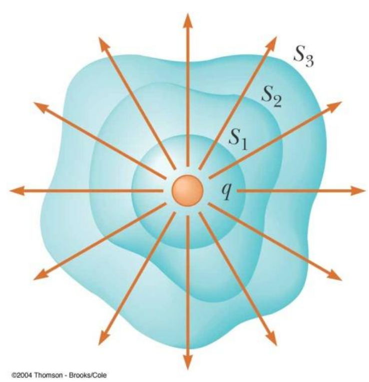
GAUSS'S LAW
To see how the net flux through a closed surface is quantitatively related to the net charge enclosed by the closed surface lets consider the flux due to a point charge.

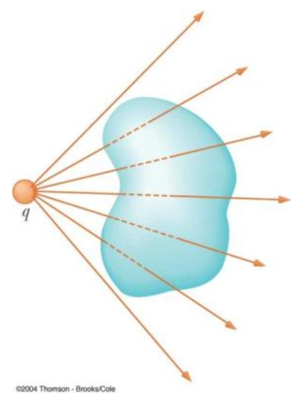
Flux Due to a Point Charge



$$\Phi = \oint \vec{E} \bullet d\vec{A} = \oint EdA\cos\theta$$
$$\Phi = \oint EdA \quad (\theta = 0^o)$$
$$\Phi = E\oint dA$$
$$\Phi = E4\pi r^2$$
$$\Phi = \frac{kq}{r^2}4\pi r^2$$
$$\Phi = 4\pi kq$$
$$\Phi = 4\pi \frac{1}{4\pi\varepsilon_o}q$$
$$\Phi = \frac{q}{\varepsilon_o}$$

Note that this result is independent of *r* and thus holds true for any spherical surface. This result holds true for any closed surface surrounding the charge because the flux must be the same.

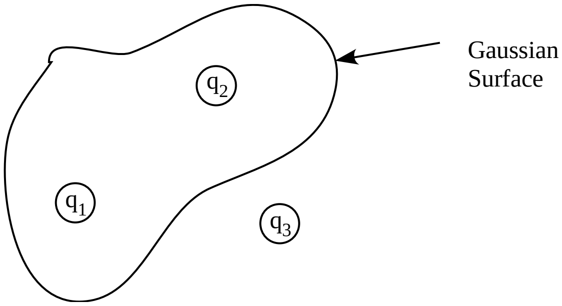




1. The net flux through any closed surface surrounding a net charge q is given by $\frac{q}{\varepsilon_o}$ and is independent of the shape of that surface.

2. The net flux through a closed surface surrounding zero net charge is zero.

Flux Due to Multiple Point Charges
Consider 3 point charges.



$$\Phi_E = \oint \vec{E} \bullet d\vec{A}$$
$$\Phi_E = \oint \left(\vec{E}_1 + \vec{E}_2 + \vec{E}_3\right) \bullet d\vec{A}$$
$$\Phi_E = \oint \vec{E}_1 \bullet d\vec{A} + \oint \vec{E}_2 \bullet d\vec{A} + \oint \vec{E}_3 \bullet d\vec{A}$$
$$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 \quad (\Phi_3 = 0)$$
$$\Phi_E = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} = \frac{(q_1 + q_2)}{\varepsilon_o}$$
$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

* $\vec{E}$ = total electric field at any point on the Gaussian surface due to all surrounding charges.

$$\boxed{\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}}$$ Gauss's Law

1. Gauss's Law provides a much more convenient method of calculating E-fields of highly symmetrical charge distributions.
2. Gauss's Law is evaluated over an imaginary closed surface called a Gaussian Surface. It should be chosen such that is has the same symmetry as the charge distribution so that $\Phi_E = \oint \vec{E} \bullet d\vec{A}$ is simple to evaluate.
3. Gauss's Law is more general than Coulomb's Law in the sense that it's always valid and Coulomb's Law is only valid for static charge distributions.
4. Gauss's Law is one of the Fundamental Maxwell's Equations.

Does zero net flux mean zero E-field??? NO!!!

Zero net flux implies two situations:

1. No charged particles enclosed within surface.
2. There are charged particles, but the net charge enclosed is zero.

Gauss's Law states that the net electric flux through any closed surface is proportional to the net charge enclosed, not the E-field!