Three basic strategies for approaching formal proofs

1. "Backward planning" – First recognize the *type* of sentence you're trying to prove (conjunction/disjunction/etc.), and then ask yourself the following two questions:
   a. Is this sentence already embedded into a previous sentence? If so, then you'll probably use an elimination rule to arrive at your target.

      Example: A&(BvC) ├ BvC

      In this case, our conclusion is a disjunction that is already embedded within the original assumption, so we'll use an elimination rule to 'pull' it out.

1	(1)	A&(BvC)	A
1	(2)	BvC	1&E

   b. If this sentence is not already embedded into a previous sentence, can you *create* the target using an introduction rule? If so, what will you need in order to apply the rule?

      Example: A&(BvC) ├ AvD

      In this case, our conclusion is a disjunction that does not occur in the original assumption. We'll try to 'create' it by using the main rule that allows us to introduce new disjunctions—the vE rule. In order to yield AvD using the vE rule, we know that we only need one of the component disjuncts. If we can get either A or D from our original assumption, then we'll be able to complete our proof. But now we have to do some more 'backward planning': Can we get either A or D from our original assumption? In this case, we *can* get A, by applying the &E rule.

1	(1)	A&(BvC)	A
1	(2)	A	1&E
1	(3)	AvD	2vI

      Backward planning often involves looking several moves ahead, but it's very frequently enough to identify the strategy for an entire proof.

2. “Break stuff up and see what happens”—If you’re unable to ‘see’ how you might derive a given sentence, but feel like it should be possible to get from your given assumptions, it’s sometimes worth simply breaking complex sentences into simpler ones. You never know—the strategy may become clearer to you after doing this.

Example: A&(B->(~Av(C<->D))), B ⊢ D->C

In this case, some folks might not be able to recognize where the conditional D->C is ultimately going to come from. That’s ok—we can still try to ‘break up’ our first premise, and see if anything useful results:

1	(1)	A&(B->(~Av(C<->D)))	A
2	(2)	B	A
1	(3)	A	1&E
1	(4)	B->(~Av(C<->D))	1 &E

Are you able to ‘see it’ using strategy #1 yet? If not, that’s ok. Just keep breaking the pieces apart using the rules available to you:

1,2	(5)	~Av(C<->D)	2,4->E

And if you’re still not ‘seeing it’, notice that we can still break this up further:

1,2	(6)	C<->D	3,5vE

At this point, our target sentence has just about revealed itself. We need only ‘pull’ it from line 6, using our <->E rule.

1,2	(7)	D->C	6<->E

Note that this strategy may sometimes result in unnecessary ‘moves’ in your proof. You might sometimes break a complex sentence up into its component parts, only to find that you don’t need all of those component parts to complete the proof. This isn’t a problem, per se. A 5 line proof is just as complete as a 4 line proof, so long as they both derive the intended conclusion resting only on given assumptions, and neither violates any of the rules of proof.

3. “If all else fails, try a reductio”—If neither of the first two strategies works, you have a ‘last resort’ available. Assume the *denial* of your target sentence, try to find a contradiction, and then use the RAA rule to arrive at your desired conclusion.

Example: P->Q, P->~Q, P&R ├ B

In this case, we’ll exhaust our first two strategies pretty quickly. The conclusion isn’t embedded in any of our given assumptions, and it doesn’t appear that any of our introduction rules will yield the atomic sentence B. So much for strategy #1. We could try breaking things up a bit, and see where that gets us:

1	(1)	P->Q	A
2	(2)	P->~Q	A
3	(3)	P&R	A
3	(4)	P	3&E
1,3	(5)	Q	1,3->E
2,3	(6)	~Q	2,4->E

That’s about as far as strategy #2 will take us, but there’s still nothing that looks like our conclusion in sight. So now we try strategy # 3. Since we want to arrive at B, let’s assume its denial, ~B:

7	(7)	~B	A

Now we look for a contradiction. Sometimes we’ll need to *derive* a contradiction, but in this case we’re in luck—the contradiction is already there for us. Lines 5 and 6 will allow us to employ the RAA rule, which will in turn allow us to discharge the assumption we made at line 7. This yields our conclusion:

1,2,3	(8)	B	5,6 RAA (7)

Just keep in mind that when you use the RAA rule, the line you discharge *must be an assumption*. The rule will not allow you to discharge derived sentences. Only *assumptions* may be discharged.